PROBABILITY APPLICATIONS IN QUANTITATIVE TRADING

MATHEMATICAL PROBABILITY METHODS

- PRINCETON QUANTITATIVE TRADERS

FOUNDATIONS OF PROBABILITY

Understanding Random Variables

Definition and Role

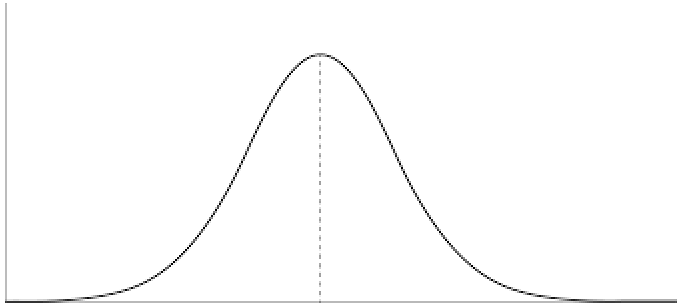
Random variables represent numerical outcomes of stochastic processes key to probability theory.

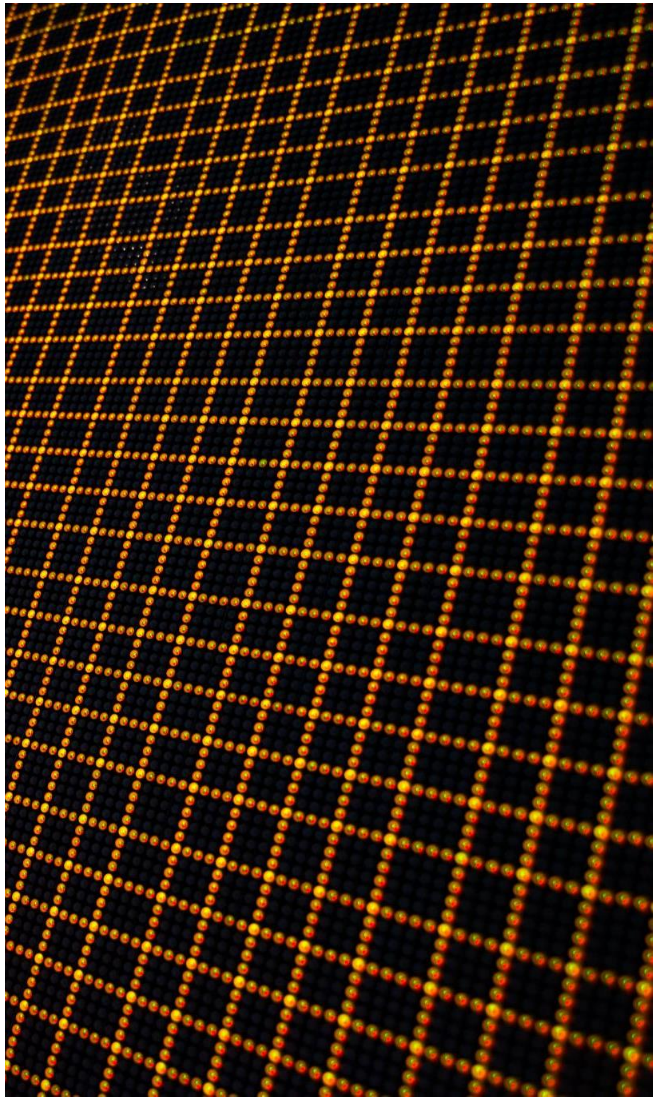
Modeling Financial Data

Random variables model uncertain financial phenomena like asset returns and price movements.

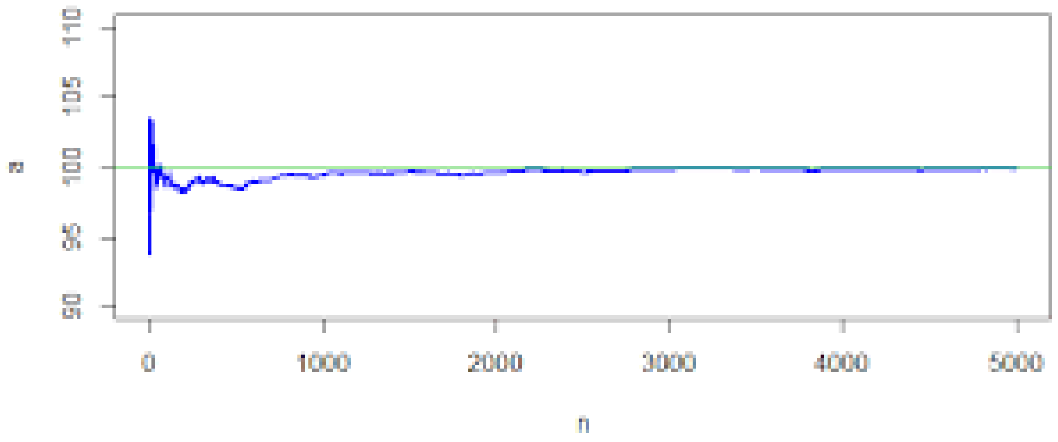
Example - Stock daily returns can be modeled as normally distributed variables with mean and variance.

- Random Variables: $X \sim N(\mu, \sigma^2)$
- Expectation: $E[X] = \int x \, f(x) \, dx$
- Variance: $Var(X) = E[(X - E[X])^2]$



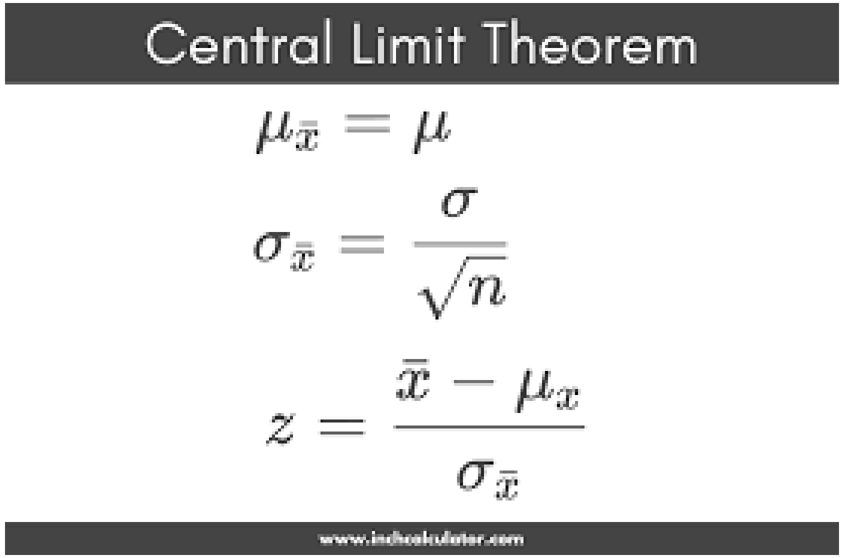


Law of Large Numbers and Central Limit Theorem



Law of Large Numbers

As trials increase, sample averages approach the expected value, validating large dataset trading strategies.



Central Limit Theorem

Sum of many independent variables tends toward a normal distribution, regardless of original data shape.

PROBABILITY TECHNIQUES

Markov's Inequality

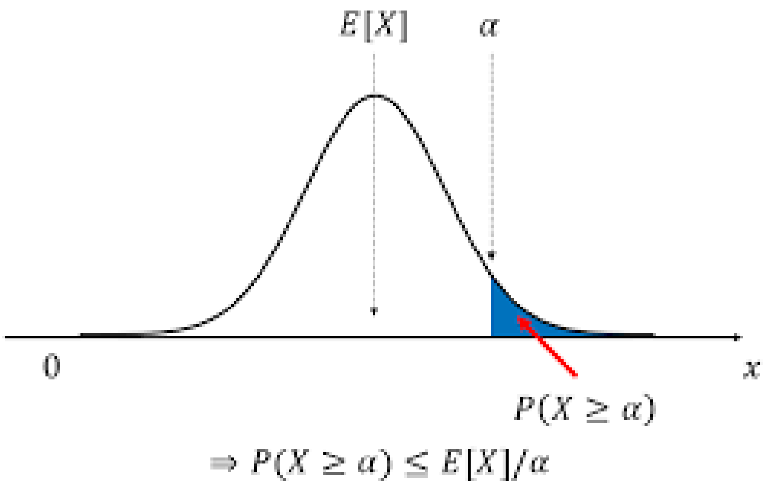
Application in Quantitative Trading

Basis for more complex probability measures. It tells us the probability of our random variable being realized above a threshold.

$$P(X \geq t) \leq \frac{E(X)}{t}$$

Unkown Distribution

Markov's Inequality works for any distribution where X is a non-negative random variable, such as price.



Derivation

Law of Total Expectation: E[X] can be expressed as a weighted average of the values below and above and threshold t.

This allows us to find a lower bound for E[X] -- imagine moving all values to the left of t to 0 and to the right of t to t

$$E(X \mid X < t) \cdot P(X < t) + E(X \mid X \geq t) \cdot P(X \geq t) = \mathbf{E(X)}$$

Chebyshev's Inequality in Risk Management – 'Special case of Markov's Inequality'

$$P(|X-\mu| \geq t) \leq \frac{Var(X)}{t^2}$$

- X ~ R.V.
- μ ~ expected value
- σ ~ standard deviation

Application in Quantitative Trading

Used to estimate extreme event likelihoods like significant drawdowns in trading strategies. Ie. the probability of being far away from the mean can't be too big

Non-parametric Probability Bounds (Including Negative R.V.s)

Chebyshev's inequality sets probability limits on deviations from the mean without relying on distribution assumptions

Setting Conservative Risk Limits

Helps traders design strategies by accounting for unknown or non-normal return distributions.

PROOF

$$P(|X-\mu| \geq t)$$

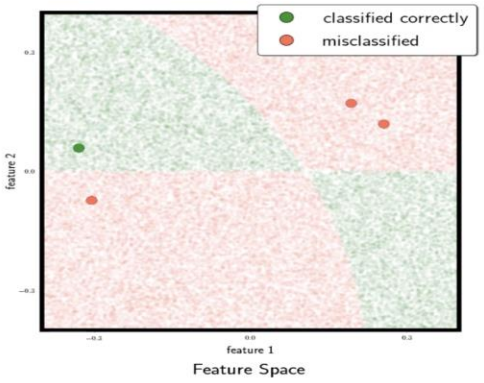
$$= P((X-\mu)^2 \geq t^2) \leq \frac{E[(X-\mu)^2]}{t^2}$$

$$= \frac{Var(X)}{t^2}$$

Hoeffding Bound

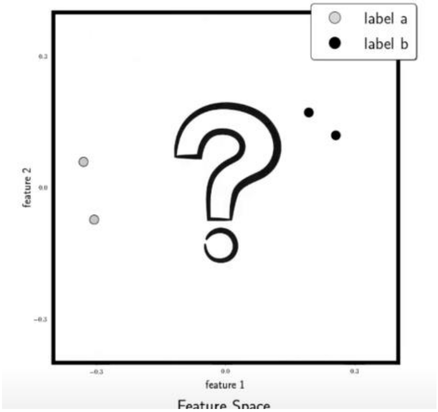
Application in Quantitative Trading

Provides an upper bound on the probability that the sum of bounded independent random variables deviate from its expected value by a threshold

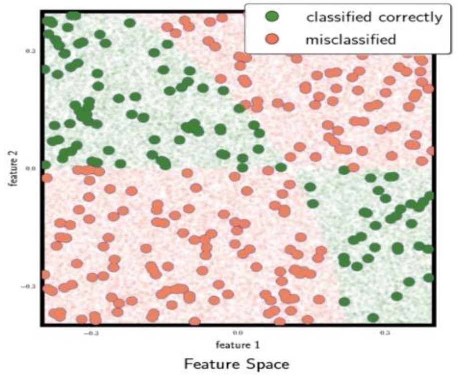


$$\mathbf{E_{train}(h)} \; = 0.7500 \quad \text{Training Error}$$
$$\mathbf{E_{true}(h)} \; = 0.6302 \quad \text{True Error}$$

Can we guess a distribution from 'n' samples?



Hoeffding Inequality gives us probability of incorrect classification given n samples



$$\mathbf{m} = 100 \quad \text{Sample size}$$
$$\boxed{\varepsilon = 0.1 \quad \text{Approximately close}}$$
$$\mathbb{P}(|\mathbf{E_{train}(h)} - \mathbf{E_{true}(h)}| > \varepsilon) \leq 2 \;\; \exp^{-2m\varepsilon^2}$$

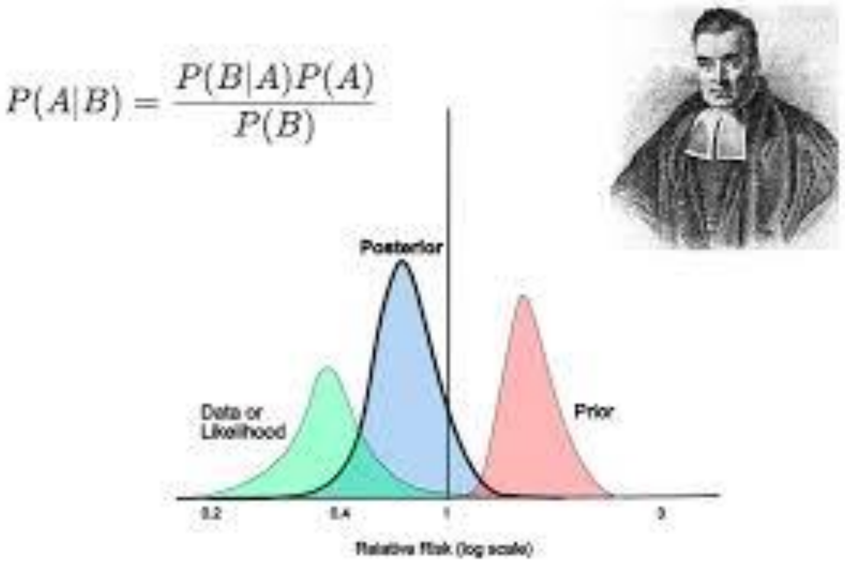
Bayesian Probability

Bayesian Updating Framework

Bayesian methods update beliefs about uncertain parameters using new observational data efficiently.

Volatility Estimation in Trading

Bayesian volatility estimation helps traders revise confidence in strategies after market changes or drawdowns.



Orderbooks using Poisson Arrival

Poisson Process in Trading

Poisson processes model the probability of order arrivals and executions in fixed time intervals

Fill Probability Formula

The fill probability of a limit order is calculated as P(fill) = 1 - e^{-λt}, where λ is the expected order arrival rate.

Trading Strategy Optimization

These models help traders optimize order placement, assess execution risk, and enhance performance in high-frequency trading.

Discord and Study Groups